Common Pitfalls in Mathematical Induction Proof

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Abstract. *Many students encounter difficulties when learning the material on mathematical induction. A lack of understanding of the principles of mathematical induction leads students to make errors when attempting to prove mathematical statements repeatedly. To help students avoid these pitfalls, it is essential to analyze common pitfalls that frequently occur in the proof process using mathematical induction. This study employed a qualitative approach, utilizing a case study design, to analyze the answer sheets of 56 students from the first semester of the Mathematics Education program. The answers were categorized into five groups based on the types of difficulties encountered. From each category, one information-rich case was selected for further interviews. The research revealed several common pitfalls in mathematical induction: (1) failure to write the complete mathematical statement to be proven, (2) misunderstanding the importance of defining the domain of numbers, (3) using examples to prove the mathematical statement, (4) pitfall due to lack of metacognitive control, and (5) performing operations on both the left and right sides of the mathematical statement in the second stage*

Keyword: *Error Analysis; Mathematical Proof; Mathematical Induction; Pitfall*

Abstrak. *Banyak mahasiswa mengalami kesulitan ketika mempelajari materi induksi matematika. Kurangnya pemahaman terhadap prinsip-prinsip induksi matematika menyebabkan mahasiswa melakukan kesalahan saat mencoba membuktikan pernyataan matematika secara berulang. Untuk membantu mahasiswa menghindari jebakan konseptual (pitfalls) tersebut, penting untuk menganalisis pitfalls umum yang sering terjadi dalam proses pembuktian menggunakan induksi matematika. Penelitian ini menggunakan pendekatan kualitatif dengan desain studi kasus untuk menganalisis lembar jawaban dari 56 mahasiswa semester pertama Program Studi Pendidikan Matematika. Jawaban-jawaban tersebut dikategorikan ke dalam lima kelompok berdasarkan jenis kesulitan yang dialami. Dari setiap kategori, dipilih satu kasus yang kaya informasi untuk diwawancarai lebih lanjut. Hasil penelitian menunjukkan beberapa pitfalls umum dalam induksi matematika, yaitu: (1) tidak menuliskan pernyataan matematika yang akan dibuktikan secara lengkap, (2) kurang memahami pentingnya menentukan domain bilangan, (3) menggunakan contoh untuk membuktikan pernyataan matematika, (4) pitfall akibat kurangnya kontrol metakognitif, dan (5) melakukan operasi pada kedua ruas kiri dan kanan pernyataan matematika pada tahap kedua pembuktian.*

Kata Kunci: *Analisis Kesalahan; Pembuktian Matematis; Induksi Matematika; Pitfall*

INTRODUCTION

Proof by mathematical induction is an abstract concept that students typically encounter for the first time in grade 11 of secondary school. Unsurprisingly, many students struggle to comprehend this unfamiliar and abstract material. This challenge is supported by the findings of Papadopoulos & Kyriakopoulou (2022), who concluded that mathematical induction is a particularly difficult concept for high school students. These difficulties often persist in higher education. Nardi & Iannone (2003) further observed that pre-service teachers also face significant challenges with number theory courses, particularly with the topic of mathematical induction.

This difficulty is also reflected in students' own statements when learning mathematical induction. For example, one student stated, "I do not understand why we must assume $P(k)$ first before proving $P(k+1)$, and the steps of induction feel confusing." Another student also remarked, "I find it difficult to write the mathematical statement completely." These statements indicate that students experience difficulties when attempting to prove statements using mathematical induction, which often leads to repeated errors in their proof attempts. Similar patterns of repeated mistakes were also observed among pre-service teachers from the Mathematics Education program at Universitas Bengkulu when working on mathematical induction tasks. Research conducted by Dogan (2016) reported a similar thing in that students have difficulties both pedagogically and cognitively when learning mathematical induction and consider that mathematical induction is a circular proof.

Errors made by students have been widely studied. Stylianides & Stylianides (2009) note that proof represents a transition from empirical arguments to formal reasoning, yet students often focus on procedures while neglecting underlying concepts in mathematical induction (MI), such as defining the universe of numbers. This tendency is influenced by prior learning habits that emphasize formula-based problem solving. Similarly, Uygun (2020) found that beginners in MI tend to rely on memorized procedures or adapt steps from similar problems without fully understanding the conceptual foundations.

Students who frequently make errors in proofs using mathematical induction are often unaware of their mistakes. They believe that their solutions are correct, even when experts point out the errors. This reflects a deep-seated misconception of the underlying concepts. Interestingly, a notable phenomenon emerges from this situation: groups of students often share the same misconceptions and consequently make similar errors. Such recurring patterns of misconceptions and errors are often discussed in the literature as "common pitfalls" in students' mathematical thinking (Khisty & Radosavljevic, 2010).

Pitfalls are traps or difficulties that we do not see, or they can be interpreted as difficulties that are hidden or not easily recognized. Khisty & Radosavljevic (2010) define a pitfall as a typically erroneous way of thinking about the problem or solution strategy. Unlike a common mistake, the person who gets a pitfall does not realize that they are caught in a mistake, even until somebody else tells them. At first, those affected by pitfalls defend their understanding until their understanding is finally corrected. The difference between error and pitfall is illustrated in the understanding quadrant (Figure 1).

	True	False
Know the reason	Understand	Mistake
Don't know the reason	Remember	Pitfall

Figure 1. The Understanding Quadrant

When someone who has made a mistake is informed of their error, they often recognize and understand what they did wrong. However, a unique phenomenon occurs with individuals caught in a pitfall—they are unable to comprehend why their reasoning is incorrect, even when it is pointed out to them. This is often the result of misconceptions formed during initial learning. Heller et al. (2007) use the concept of pitfalls in mathematics learning as a foundation for inquiry and deeper understanding. Additionally, Thompson & Rubenstein (2000) noted that many mathematical vocabulary terms can also lead to pitfalls, further complicating the learning process.

Baker (1996) identified nine difficulties that cause student errors when dealing with the concept of mathematical induction. The nine difficulties are (a) mathematical resources, (b) conceptual understanding, (c) convincing evidence, (d) everyday reasoning, (e) metacognitive control, (f) heuristics, (g) meaningfulness, (h) procedural knowledge, (i) affective factors. Baker (1996) states more specifically that students have difficulty with the mathematical content of induction, especially with the inability to operate with symbols. They rely exclusively on procedures that lack conceptual understanding. They rely heavily on examples to recognize that something is proven. They show a lack of metacognitive skills. Building on Baker's framework of difficulties, this study aims to examine how these learning difficulties manifest as cognitive traps or "pitfalls" in students' attempts to construct proofs using mathematical induction.

Previous research always focuses on the errors that occur. Research conducted by Handayani (2021); Fitriani et al. (2021), Ernawati & Ilhamuddin (2020); Bani & Ate (2020); Rusdiantoro (2020); Nardi & Iannone (2003) and Stylianides et al. (2017) who describe all errors that appear in their samples. However, most of these studies primarily describe the types of errors that occur, rather than examining whether these recurring errors function as cognitive traps or "common pitfalls" in students' reasoning. Therefore, further investigation is needed to understand which errors repeatedly trap students when applying the concept of mathematical induction. Based on this background, this study aims to identify and analyze common pitfalls that occur when pre-service mathematics teachers construct proofs using mathematical induction.

METHOD

This study employs a qualitative descriptive research design using a case study approach. Qualitative descriptive research aims to systematically describe and interpret a phenomenon based on empirical data. The case study approach was selected because it enables an in-depth investigation of a particular phenomenon within its real-life context. According to Yin (2018), a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, particularly when the boundaries between the phenomenon and the context are not clearly evident.

In this study, the case examined is the occurrence of common pitfalls experienced by students when constructing mathematical proofs using mathematical induction. By employing a case study approach, this research aims to obtain an in-depth understanding of how these pitfalls emerge in students' reasoning during the proof process. The subjects of this study were 56 first-semester undergraduate students of Mathematics Education at Universitas Bengkulu in 2022 who were enrolled in a Mathematical Induction course. The research instrument consisted of a test comprising 10 essay problems requiring students to construct proofs using mathematical induction. The problems involved statements in the form of equalities, inequalities, and divisibility. The problems were not designed with different levels of difficulty. Prior to administration, the instrument was validated by an expert in mathematics education.

The data analysis in this study employed a qualitative descriptive approach that involved analyzing students' written responses and interview data. The overall research procedure is summarized in the flowchart presented in Figure 2. The analysis began by examining all students' responses and classifying them into correct and incorrect answers. Students who provided incorrect answers were subsequently interviewed to determine the pitfall understanding quadrants, that is, to identify whether the errors represented mistakes or deeper pitfalls in their reasoning during the proof process. During the interviews, students were asked guiding questions such as: "Are you confident with your proof steps?", "Your proof process is incorrect. Do you know where the error occurred?", and "In this part you made a mistake. Can you explain why this step is incorrect?"

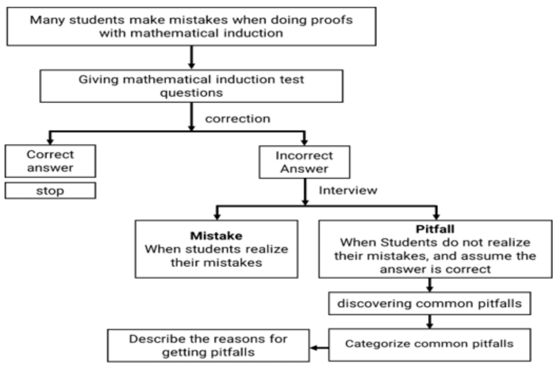

Figure 2. Research Flow Chart

After the discovering pitfall process was conducted and the identified pitfalls were categorized according to the difficulty categories proposed by Baker (1996), one subject from each category was selected purposively. The selection criteria included students who demonstrated good communication skills and provided information-rich explanations. These selected subjects were then interviewed more deeply to confirm their reasoning processes when constructing the proofs and to explore the underlying reasons for the occurrence of the pitfalls. To ensure the credibility of the findings, triangulation was carried out through member checking, in which the researcher confirmed the interpretations of the data with the participants. Based on the analysis of students' written responses and interview results, the characteristics of the identified pitfalls and the factors contributing to their occurrence in the process of constructing mathematical induction proofs were subsequently described within each category.

RESULT

A pitfall is a cognitive trap that leads learners to make errors. The key distinction between a pitfall and a simple mistake lies in the learner's conceptual understanding. Learners caught in a pitfall are often highly confident in their misconceptions, leading them to reject or resist corrections. Common responses from such learners include phrases like, "I'm sure this is correct," "This aligns with the concept," or "Really? It doesn't seem wrong."

Pitfalls are common in mathematical induction proofs, where students struggle to distinguish between valid proofs and invalid arguments (Stylianides et al., 2017). Analysis of written responses and interviews identified five recurring pitfalls, reflecting difficulty categories proposed by Baker (1996). However, categories such as everyday reasoning, heuristics, meaningfulness, and affective factors were not observed and thus excluded. Table 1 summarizes the conceptual correspondence between Baker's framework and the identified pitfalls.

Table 1. Relationship between Baker's (1996) Difficulty Categories and the Pitfalls Identified in This Study

Baker's Difficulty Category	Description in Baker (1996)	Corresponding Pitfall in This Study
Mathematical resources	Difficulty in manipulating mathematical symbols and expressions	Pitfall in operating mathematical symbols/content
Conceptual understanding	Incomplete understanding of the concept of induction	Pitfall in interpreting the principle of mathematical induction
Convincing evidence	Belief that examples are sufficient to justify a general statement	Pitfall in proof by example
Metacognitive control	Lack of awareness in evaluating reasoning processes	Pitfall due to lack of metacognitive control
Procedural knowledge	Overreliance on memorized procedures without conceptual understanding	Pitfall in mathematical procedural understanding

Several other categories identified by Baker (1996), such as everyday reasoning, heuristics, meaningfulness, and affective factors, were not explicitly observed in the present dataset. Therefore, they are not represented as separate pitfall categories in this study.

Research has identified five common pitfalls in mathematical induction proof procedures: (1) pitfalls in operating mathematical symbols or content: Errors in manipulating symbols or mathematical expressions, (2) pitfalls in interpreting the principle of mathematical induction: Misunderstanding or misapplying the fundamental principle, (3) pitfall in proof by example: Believing that proving a concept with specific examples is sufficient for general proof, (4) pitfall due to lack of metacognitive control: failing to monitor and evaluate one's reasoning process, (5) pitfall in mathematical procedural understanding: Relying on rote procedural steps without grasping the underlying concepts. A more detailed discussion of these five pitfalls is presented in the following section

1. Pitfall of operating mathematical symbols/content

A common pitfall in students' proof processes involves errors in operating mathematical symbols or content. Research findings indicate that this pitfall often manifests as students failing to write mathematical statements accurately or completely. Specifically, students frequently neglect to specify the universe of the set of numbers being discussed. For instance, they may omit writing k(1), k∈N, when defining P(k), or they might incorrectly default to writing k∈R due to a habitual focus on real numbers. Sutini (2019) similarly highlighted in her research that students often overlook the proper use of symbols, particularly in the context of set theory. These errors reflect a lack of attention to detail and a fundamental misunderstanding of the role of precise notation in mathematical proofs.

1.1 Did not write P(k) completely

The pitfall of operating on mathematical symbols/content is not being able to write mathematical statements correctly.

Question: *Use mathematical induction to prove that* $\sum_{k=1}^{n}\ \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}\leq 2-\frac{1}{n}$ *whenever n is a positive integer.*

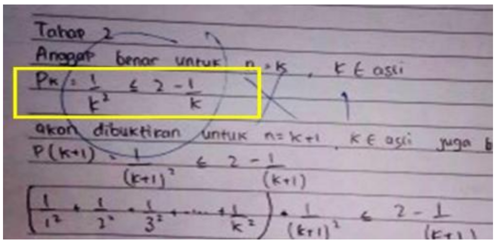

Figure 3. Common Pitfalls Not Write P(k) completely

Based on Figure 3, it can be seen that the subject was unable to write mathematical statements correctly. In the part marked with a box, the subject should have written P(k) completely, such as "P(k): $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{k^2}\leq 2-\frac{1}{k};\ k\,\epsilon\, N$". The subject misunderstood that P(k) only changes n with k without writing the sequence completely. Based on information from the interview, in working on this problem the subject stated did not understand the material about mathematical induction.

1.2 Writes left-hand side and right-hand side operation not separately

The second type of pitfall of operating mathematical symbols/content is not writing the right-hand side and left-hand side operations separately. The pitfall can be seen in Figure 4.

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

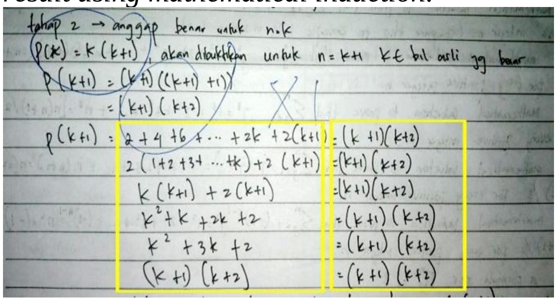

Figure 4. Common Pitfall of write left-hand and right-hand operation not separately

Based on Figure 4, it can be observed that the subject did not distinguish between the operations on the left-hand side and the right-hand side. The subject only performed operations on the left-hand side without making any modifications to the right-hand side. During the interview, the subjects explained that they approached mathematical induction in this manner because it mirrored the methods they had been taught in high school. Therefore, it can be concluded that the subject in Figure 4 was capable of performing mathematical manipulations accurately but lacked a comprehensive understanding of mathematical induction, relying instead on procedural habits developed in high school.

1.3 The Number Universe is not Written

The third type of pitfall in the operation of mathematical symbols/contents is not writing down the universe of numbers. An example of a subject who gets a pitfall of not writing down the universe of a number is shown in Figure 5.

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

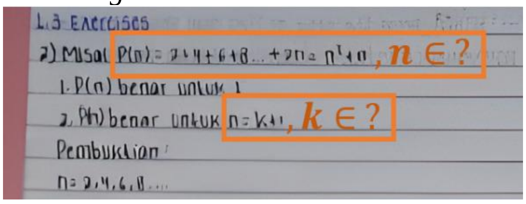

Figure 5. Common Pitfall Not Writing down the Number Universe

Most students who encountered pitfalls in this context tended to underestimate the importance of writing the symbol for the number universe. During the interview, the subject stated that the number universe was not considered significant because, in their high school experience, it was not emphasized during lessons on mathematical induction. The focus was primarily on performing the manipulations in the induction step correctly. This indicates that the subject did not fully grasp the role or significance of defining the number universe in the proof process. The omission highlights the critical role of the number universe in mathematical induction. Without a clear definition of the numbers involved at each stage, the "domino effect" analogy central to induction cannot function properly. Furthermore, he suggests incorporating visual aids to help students better comprehend this concept.

2. The pitfall of interpreting the principles of mathematical induction

There is a common pitfall, namely students who cannot interpret the principles or steps of mathematical induction. The results showed that the common pitfalls were (1) assuming the proof process is correct at the first stage, (2) only writing the operation on the right-hand side, (3) not elaborating P(k) and/or P(k+1), (4) ignoring the writing of the conclusion of the proof, and (5) not using the correct induction concept when writing P(1). These pitfalls can be seen as follows.

2.1 Assuming the proof process is correct at the first stage.

The first type of pitfall that occurs in the principle of Mathematical Induction (MI) is assuming the proof process is correct at the first stage. Where the first stage in mathematical induction is to show P(n) is true for n=1. It can be seen in Figure 6 that subjects are getting pitfalls in this category.

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

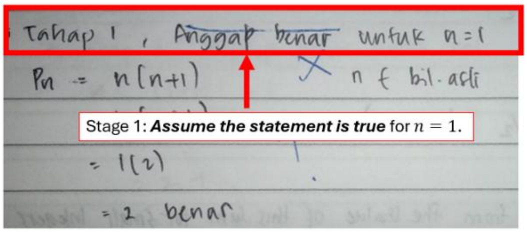

Figure 6. Common Pitfall of Assuming the Proof Process is Correct at the First Stage

P(1) is the first step in mathematical induction that should be verified. From Figure 6, it can be seen that students ignored the P(1) step and wrote "assume true for n=1". Students should have verified the correctness of P(1) and then proceeded to the second step. When confirmed through interviews, the subjects stated that they did not understand the information on each of the induction steps.

2.2 Write/operate the right-hand side only

In the second stage of mathematical induction, it begins by writing the statement P(k), which is a complete mathematical statement consisting of the Left-Hand Side and Right-Hand Side. Then continued by writing the complete P(k+1) statement (left and right). In this study, there was a pitfall where the subject made P(k) and P(k+1) statements by only writing the right-hand side statement.

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

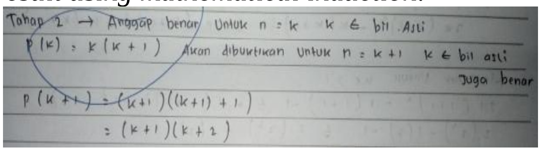

Figure 7. Common Pitfall of Only Writing the Right Segment in P(k) or P(k+1)

Figure 7 illustrates that in stage 2 of the proof, the subject wrote P(k) incompletely, including only the right-hand segment of the statement. Similarly, for P(k+1), the mathematical expression was also written with only the right-hand segment. The subject argued that it was unnecessary to write the complete left-hand segment of P(k) and P(k+1) because the full statement was already provided in the problem. This suggests that the student lacks a proper understanding of how to correctly express P(k) and P(k+1) in the proof.

2.3 Did not Write the Statements P(k) and/or P(k+1) in Second Stage

The third pitfall in Mathematical Induction is not writing the statement P(k) and/or P(k+1) in the second stage. At this stage, P(k) and/or P(k+1) should be written completely in the proof stage. The pitfall can be seen in Figure 8 below.

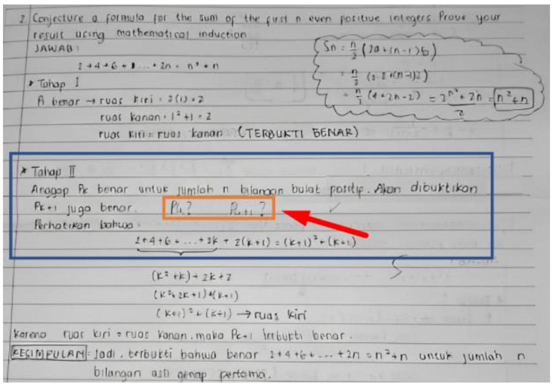

Figure 8. Common Pitfall of Not Writing the P(k) and/or P(k+1) Statement at Second Stage

Students often do not write the statement P(k) and or P(k+1) in stage 2 for the reasons: 1) it is not important to write it completely because the proof is most important, 2) using the method in high school, and 3) not careful. Students provided "descriptive definitions" where the mathematical induction principle was explained in a personalized way, as steps to follow, or a formal type by recalling textbook statements.

2.4 Ignoring Writing the Final Conclusion of the Proof

In mathematical induction, the conclusion is the final statement that summarizes the proof and shows that the desired result applies to all values of the variable in question. So with the conclusion, it can be understood that all numbers included in the universe of the statement are true if substituted into the statement P(n).

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

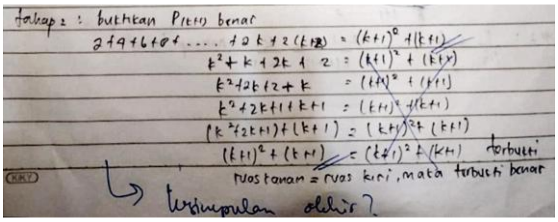

Figure 9. Common Pitfall of Not Writing the Final Conclusion

Figure 9 reveals that the subject did not write the final conclusion of the proof. The conclusion should have been stated as follows: "For all positive integers n, the sum of the first n even numbers is equal to $n^2 + n$."

2.5 Did not use the correct induction concept when writing P(1)

Students are getting pitfall with a wrong understanding of the steps in mathematical induction proof. Like the subject in Figure 10 who wrote step 1 of the mathematical induction proof by "assuming true" P(1). Supposedly, P(1) is proven true directly, where P(1) is a statement when the value of n is 1. Harel (2024) said that the first step taken in a proof using mathematical induction is to prove P(1) true.

Question: *Conjecture a formula for the sum of the first n even positive integers. Prove your result using mathematical induction.*

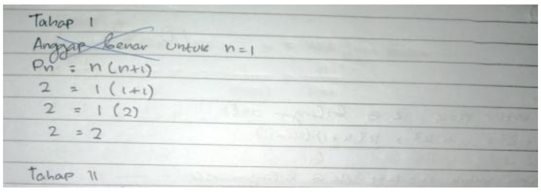

Figure 10. Common Pitfall of Not Writing P(1) Correctly

Firstly, it can be observed that students do not correctly understand how to write P(1) as a mathematical statement. Secondly, students fail to separate the right and left segments of the proof. A similar error was highlighted in the research by Ernawati & Ilhamuddin (2020). In Figure 10, students should have written P(1) as a mathematical statement: P(1): 2 = 1(1+1), and then separately described the left segment as 2 and the right segment as 1(1+1)=2. This would demonstrate that the right and left segments are equal. Thirdly, the student misinterprets the principle of induction, believing that P(1) "must be assumed to be true" rather than "shown to be true." During the interview, the subjects explained that they felt the steps of mathematical induction were essentially the same, with the main goal being to prove that the two segments were equal.

3. Rely on Examples

There is a common pitfall, namely students who rely on examples in proving mathematical statements in general. The results show that the common pitfall is using case examples to perform induction proofs. For example, the following student's answer.

Question: *Conjecture a formula for A^n where* $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. *Prove your conjecture using mathematical induction*

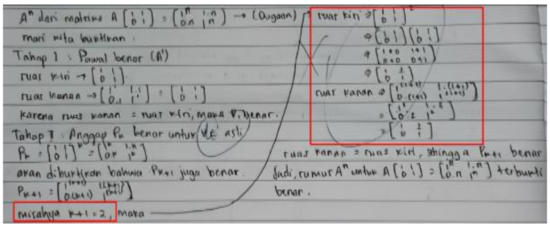

Figure 11. Common Pitfalls Using Examples to Prove Induction

Frequently, when students struggle with proof ideas, they resort to examples to demonstrate that a statement holds under specific conditions. The interview with this subject revealed that he believes that if one condition can be proven true, then other similar conditions will also hold, leading to a general conclusion based on the example.

4. Pitfall Due to Lack of Metacognitive Control

One common pitfall identified in this study is the pitfall due to lack of metacognitive control, where students fail to monitor and evaluate their reasoning strategies during the proof process. Some researchers have explained metacognition in mathematics education as successful mathematical performance. Metacognition is defined as a condition when a person is aware of tactics and strategies in his cognition in solving a problem. The results of the study show that many students are getting pitfalls when looking for the best tactic in manipulating the form of the P(k) statement so that it is proven.

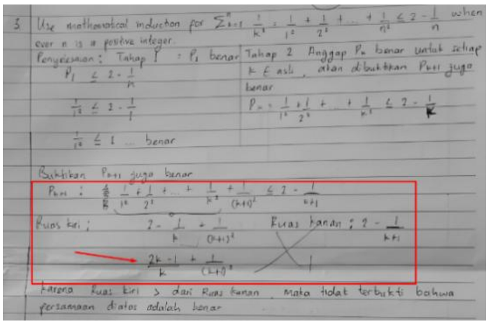

Figure 12. The common pitfalls on manipulating proven mathematical statements

Figure 12 illustrates that the subject encountered a pitfall in determining the approach to proving a given mathematical inequality. During the interview, the subject explained that while attempting to identify the correct pattern, they were unable to derive a general form for a formal conclusion.

5. Do Not Understand Mathematical Procedures

Proving an equation by carrying out the process of manipulating both sides at once is a common pitfall for students because they do not understand mathematical procedures. Figure 13 below is an example of a subject who is getting pitfall in the wrong mathematical procedure. Directly, the subject carries out the process of calculating the left and right sides of the equation that they want to prove is the same, then states that it is proven in the last line of the arithmetic operation.

Question: *proof P(n):* $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

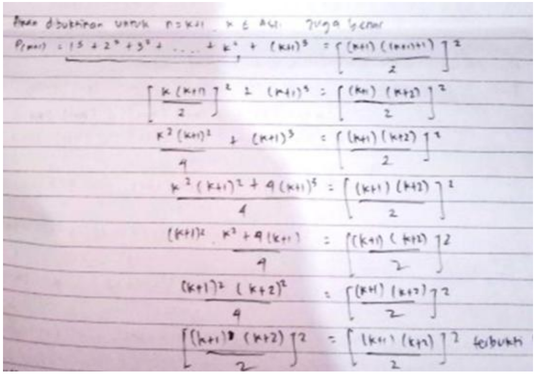

Figure 13. The common pitfall does not separate the operation of the left-hand and right-hand side

During the interview, the subject defended their approach, stating that the mathematical procedures were based on the knowledge gained in secondary school. This led to pitfalls in using incorrect mathematical procedures, as their mathematical beliefs were based on prior, incorrect knowledge. Ideally, the proof by mathematical induction, both in the first and second steps, requires separating the operations on the left-hand side and right-hand side.

DISSCUSSION

The findings indicate that errors in mathematical induction are not merely procedural mistakes but reflect deeper conceptual misunderstandings, consistent with the notion of pitfalls as cognitive traps rooted in flawed beliefs (Selden & Selden, 2003). Students often showed strong confidence in incorrect reasoning, highlighting a gap between formal concepts and their understanding. The five identified pitfalls align with Stylianides et al. (2017) and can be systematically explained through Baker's (1996) framework, showing that difficulties in induction involve interconnected aspects of conceptual understanding, procedural knowledge, and metacognitive control .

A key pattern is the dominance of procedural thinking over conceptual understanding. Students frequently performed symbolic manipulations without fully understanding the structure of induction, such as incorrectly formulating $P(k)$, omitting $P(1)$, or failing to distinguish between steps in the proof. This supports findings that students may follow procedures correctly while lacking conceptual insight (Dubinsky & Lewin, 1986; Harel, 2024). Additionally, reliance on examples to justify general statements reveals a misunderstanding of proof, consistent with Stylianides & Stylianides (2009).

Moreover, the study highlights the role of metacognitive limitations and prior learning habits. Students struggled to evaluate their reasoning and select appropriate strategies, which aligns with research emphasizing the importance of metacognition in proof construction (Öztürk & Kaplan, 2019; Weber, 2001). Their reliance on previously learned procedures, even when incorrect, further reinforces these pitfalls. Therefore, improving students' understanding of mathematical induction requires instructional approaches that emphasize conceptual reasoning, reflective thinking, and explicit understanding of proof structure rather than mere procedural execution.

CONCLUSION

This study identified five common pitfalls experienced by first-year mathematics education students when constructing proofs using mathematical induction: (1) failing to write the complete mathematical statement to be proven, (2) misunderstanding the importance of defining the number domain, (3) relying on examples to justify general statements, (4) pitfalls due to lack of metacognitive control, and (5) performing operations on both sides of the mathematical statement during the inductive step. These findings indicate that students' difficulties in mathematical induction are not only procedural but also related to conceptual understanding and metacognitive awareness in evaluating proof strategies.

This study is limited to students from a single cohort at one institution, which may restrict the generalizability of the findings. Pedagogically, the results suggest that instruction should emphasize conceptual understanding, explicit proof strategies, and opportunities for students to reflect on their reasoning processes. Future research may explore instructional approaches that support students' metacognitive.

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